The Greek letters indicate the equality of certain angles, and will assist the reader in the demonstration.

Also solved by G. W. Greenwood.

The following contributors sent in solutions to this department too late for credit in the last issue: G. B. M. Zerr solved 245; Theodore Linquist, 248 and 249; A. H. Holmes, 248, 249, and 250.

253. Proposed by SAM I. JONES, Gunter Bible College, Gunter, Texas.

The number of cubic inches contained by two equal opposite spherical segments, together with the number of cubic inches contained by the cylinder included between these segments, is 600; if this be $\frac{2}{3}$ of the number of cubic inches contained by the whole sphere, find the height of the cylinder.

Solution by THEODORE LINQUIST, Wahpeton, N. Dak.; G. W. GREENWOOD, M. A. (Oxon), Lebanon, Ill., and A. H. HOLMES, Brunswick, Me.

Let R=the radius of the sphere, and $2h$ the altitude of the cylander. Then $R-h$=the altitude of the segment of the sphere, and $\sqrt{(R^2-h^2)}$ is the radius of the base of the segment and the radius of the cylinder.

The volume of the two segments$=2[\frac{1}{6}\pi(R-h)^3+\frac{1}{2}\pi(R-h)(R^2-h^2)]$, and the volume of cylinder$=2\pi h(R^2-h^2)$.

$\therefore \frac{4}{3}\pi(R^3-h^3)$=the volume of the segments, and the cylinder$=\frac{2}{3}(\frac{4}{3}\pi R^3)$, by the conditions of the problem.

$\therefore 3h^3=R^3$. $\therefore \frac{4}{3}\pi(R^3-h^3)=\frac{8}{3}\pi h^3=600$, by the conditions of the problem.

$\therefore 2h=2\sqrt[3]{(225/\pi)}$.

Also solved by J. Scheffer.

CALCULUS.

191. Proposed by J. E. SANDERS, Hackney, Ohio.

A fly goes along a radius of a moving carriage wheel from center to circumference while the wheel makes n revolutions. If each move uniformly, what is the equation to the curve described by the fly in space, and what is its length when the wheel has made $1/m$ of a revolution?

Solution by G. W. GREENWOOD, M. A. (Oxon), Professor of Mathematics, McKendree College, Lebanon, Ill.

Take the path of the center of the wheel as x-axis, and the initial point as origin. Let the fly move on a radius making, initially, an angle ϕ with this axis. Denote the radius by a. Let C be the position of the center of the wheel, and P be that of the fly after the wheel has turned through an angle ω. Then

$$OC=a\omega,\ CP=\frac{a\omega}{2n\pi},$$

and the coördinates of the position of P are

$$x=a\omega\left(1+\frac{\cos(\omega+\phi)}{2n\pi}\right),\ y=\frac{a\omega\sin(\omega+\phi)}{2n\pi}.$$

*** An excellent solution was received from Professor Zerr. He takes the horizontal line on which the wheel travels as the x-axis, and gets for the equation of the path of the fly,

$$x=a\theta\left(1-\frac{\sin\theta}{2n\pi}\right),\quad y=a\left(1-\frac{\theta\cos\theta}{2n\pi}\right), \text{ for required length.}$$

$$s=a\int_0^{2\pi/m}\sqrt{1+\frac{1+\theta^2}{4\pi^2n^2}-\frac{\sin\theta+\theta\cos\theta}{\pi n}}\,d\theta. \qquad \text{F.}$$

192. Proposed by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Show that the volume V of the hyper-ellipsoid with semi-axes a_1, a_2, a_3, a_4, etc., in space of $2n$ and $2n+1$ dimensions is

$$V_{2n}=\frac{a_1.a_2.a_3\ldots\ldots a_{2n}.\pi^n}{1.2.3.4.5\ldots\ldots n};\qquad V_{2n+1}=\frac{2^{n+1}.a_1.a_2.a_3\ldots\ldots a_{2n+1}.\pi^n}{1.3.5.7.9\ldots\ldots(2n+1)}.$$

Solution by the PROPOSER.

Let $\left(\frac{x_1}{a_1}\right)^2+\left(\frac{x_2}{a_2}\right)^2+\left(\frac{x_3}{a_3}\right)^2+\ldots\ldots+\left(\frac{x_r}{a_r}\right)^2=1$ be the equation to the hyper-ellipsoid. Then its volume is $V=2^r\int\int\int\ldots\ldots dx_1\,dx_2\,dx_3\ldots\ldots dx_r$.

Let $x_1/a_1=y_1$, $x_2/a_2=y_2$, ..., $x_r/a_r=y_r$.

$\therefore V=2^r a_1a_2a_3\ldots\ldots a_r\int\int\int\ldots\ldots dy_1\,dy_2\,dy_3\ldots\ldots dy_r$, subject to the condition, $y_1^2+y_2^2+y_3^2+\ldots\ldots+y_r^2=1$.

$$\therefore V=\frac{a_1a_2a_3\ldots\ldots a_n[\Gamma(\frac{1}{2})]^r}{\Gamma(1+\frac{1}{2}r)}.$$

When $r=2n$,

$$V=\frac{a_1a_2a_3\ldots\ldots a_{2n}\,\pi^n}{1.2.3.4\ldots\ldots n}.$$

When $r=2n+1$,

$$V=\frac{a_1a_2a_3\ldots\ldots a_{2n+1}\,\pi^n\,\Gamma(\frac{1}{2})}{\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}\ldots\ldots\frac{2n+1}{2}\Gamma(\frac{1}{2})}=\frac{2^{n+1}a_1a_2a_3\ldots\ldots a_{2n+1}\,\pi^n}{1.3.5.7.9\ldots\ldots(2n+1)}.$$

193. Proposed by F. P. MATZ, Sc. D., Ph. D., Reading, Pa.

Find the eccentricity of the maximum semi-ellipse inscribed in a given isosceles triangle.

I. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va., and J. SCHEFFER, Hagerstown, Md.

Let the mid-point of the base be the origin, a=altitude, b=base of triangle. Let $x^2/m^2+y^2/n^2=1$ be the ellipse. Then πmn=maximum.

$\therefore n/m=y/x$.